Simple Harmonic Motion ↑↓↑↓↑↓↑↓↑↓↑↓

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February 12, 2020

1 Aim

The aim of this lab is to develop a mathematical model of the SHM of an oscillating mass on a spring.

2 Apparatus

See lab sheet.

3 Procedure

See lab sheet.

4 Data Collection

See Section 5 - Data Analysis.

5 Data Analysis

Equation Definition

$$y = A\cos(Bx + C) + D \tag{1}$$

where

y [m]: the displacement in vertical position from the equilibrium point

A [m]: the amplitude of the oscillation

B [$\mathrm{N\,m^{-1}}$]: the spring constant of the spring

x [s]: the time

C: the phase of oscillation

D [m]: the equilibrium point

5.1 Solving Variables

In Equation 1 the variables can be fitted manually by calculating each variable separately and use the real definition of each value to get the final result of the values. The manual fit result has been graphed as Figure 1

Fitting D Variable D is the displacement offset of the whole formula, and can be simply set equal to

$$D = 0.0$$

due to the detector has been zeroed correctly.

Fitting A Variable A is the amplitude of the wave and can be calculated using $\frac{\text{Max.}-\text{Min.}}{2}$ which in this case is

$$A = \frac{0.1178 - (-0.1043)}{2} = 0.1111$$

Fitting C Variable C is the phase of the wave, as known as the time offset of the wave. After tweaking with the value I got

$$C = -0.98$$

Fitting B Variable B is the frequency of the wave. After tweaking with the value I got

$$B = 8.6$$

So at last I got equation

$$y = 0.1111\cos(8.6x - 0.98) \tag{2}$$

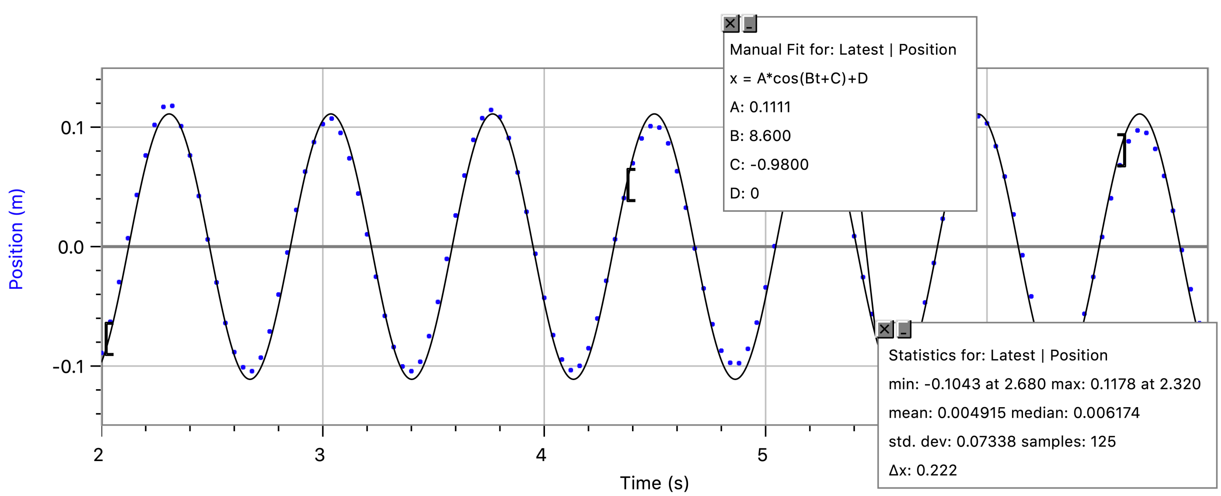
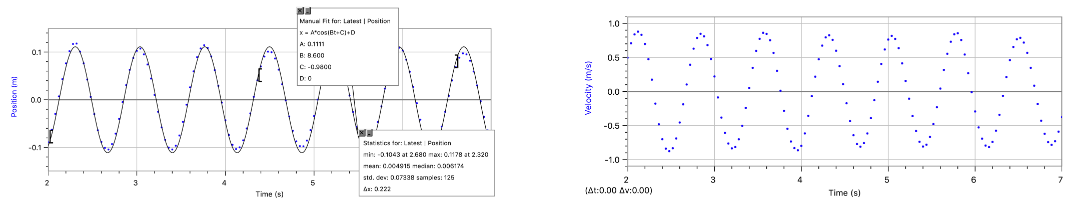


Figure 1: Time vs. Displacement (fit line: $y = 0.1111\cos(8.6x - 0.98)$)

5.2 Rate of Change

5.2.1 Velocity and Displacement



(a) Displacement vs. Time

(b) Velocity vs. Time

Figure 2: Velocity and Displacement

When the speed is at the maximum, the position is at 0.0; When the speed is zero, the mass is at 0.1111 which is the amplitude of the wave. The slope of tangent line of y graph equals to the value in the v graph, as $v = \frac{dy}{dt}$.

5.2.2 Modelling the Velocity

$$y = A\sin(Bx + C) + D \tag{3}$$

Fitting D Variable D is the position (y) offset of the whole formula, and can be simply set equal to

$$D = 0.0$$

Fitting A Variable A is the amplitude of the wave. By using the 'Statistics' tool provided in Logger Pro, we can calculate the amplitude by using $\frac{\text{Max.}-\text{Min.}}{2}$, which in this case is

$$A = \frac{0.8782 - (-0.8760)}{2} = 0.8761$$

Fitting C Variable C is the phase of the wave, as known as the time offset of the wave. We all know that sin and cos functions are completely out of phase, which then we can calculate out the phase of the wave by using

$$C = -0.98 + \pi = 2.16$$

Fitting B Variable B is the frequency of the wave. The frequency of the wave should have no difference at all, which means

$$B = 8.6$$

Which, at last, gives equation and graph as

$$y = 0.8761\sin(8.6x + 2.16) \tag{4}$$

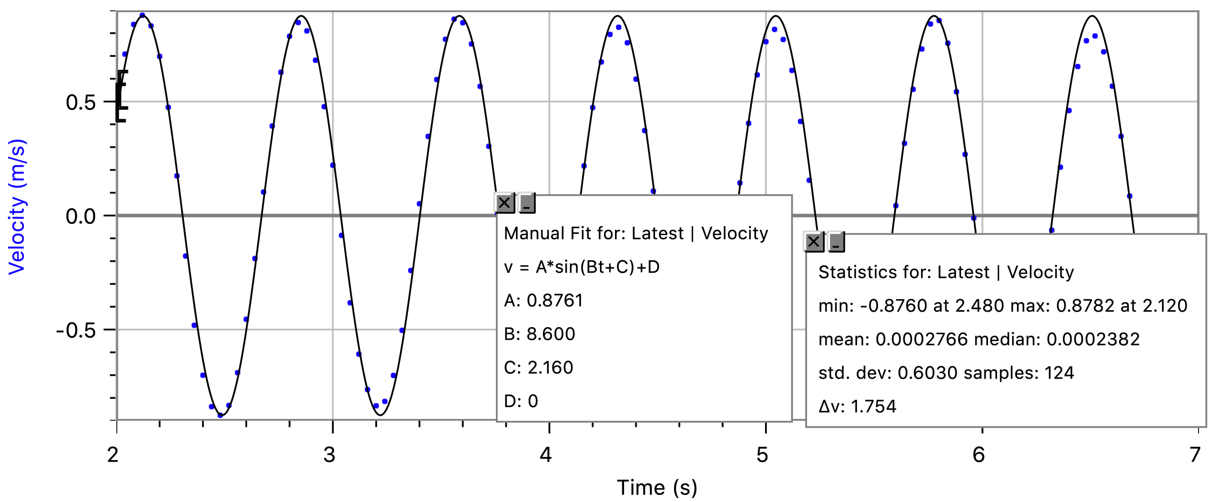
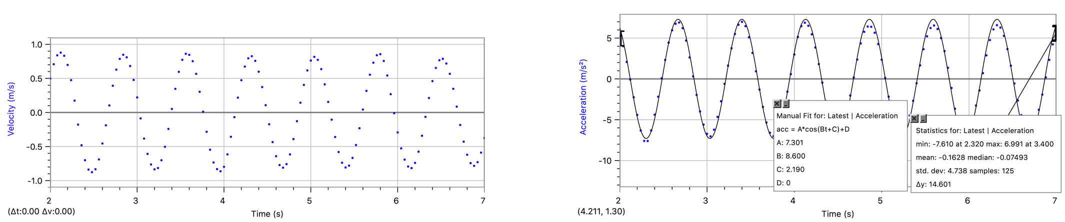


Figure 3: Time vs. Displacement (fit line: $y = 0.8761 \sin(8.6x + 2.16)$)

We can observe from the graph that the result of auto fit and manual fit is very close to each other (both the value and the appearance of the graphing line), suggesting our calculation is robust and correct.

5.2.3 Acceleration and Velocity



(a) Velocity vs. Time

(b) Acceleration vs. Time

Figure 4: Velocity and Acceleration

The slope of tangent line in *Velocity vs. Time* graph equals to the value in the *Acceleration vs. Time* graph, as $a = \frac{dv}{dt}$.

5.2.4 Modelling the Acceleration

$$y = A\cos(Bx + C) + D \tag{5}$$

Fitting D Variable D is the position (y) offset of the whole formula, and can be simply set equal to

$$D = 0.0$$

Fitting A Variable A is the amplitude of the wave. By using the 'Statistics' tool provided in Logger Pro, we can calculate the amplitude by using $\frac{\text{Max.}-\text{Min.}}{2}$, which in this case is

$$A = \frac{6.991 - (-7.610)}{2} = 7.301$$

Fitting C Variable C is the phase of the wave, as known as the time offset of the wave. We all know that sin and cos functions are completely out of phase, which then we can calculate out the phase of the wave by using

$$C = -0.98 + \pi = 2.16$$

Fitting B Variable B is the frequency of the wave. The frequency of the wave should have no difference at all, which means

$$B = 8.6$$

Which, at last, gives equation and graph as

$$y = 7.301 \sin(8.6x + 2.19) \tag{6}$$

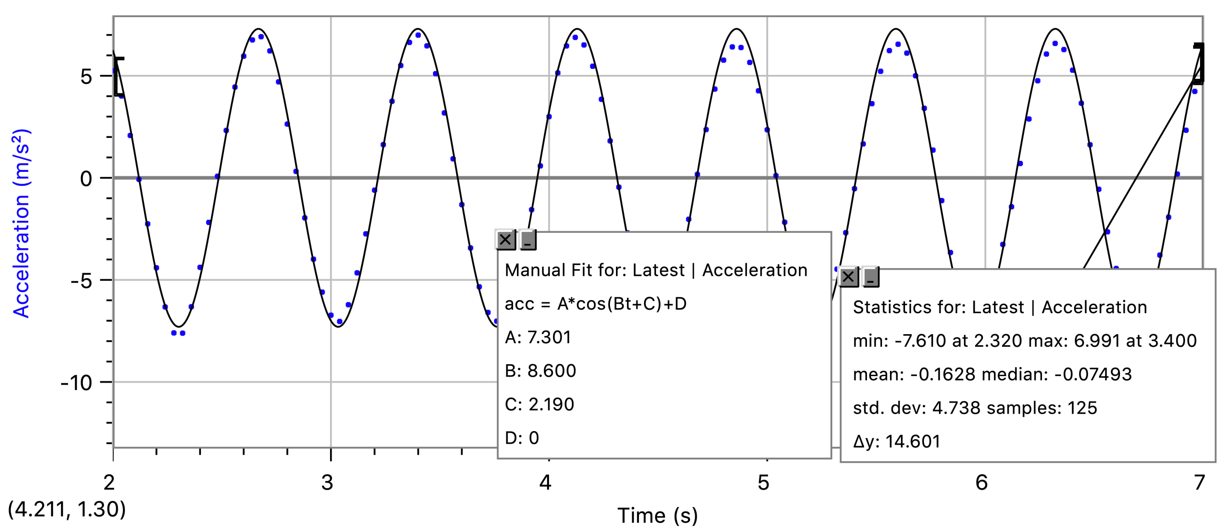
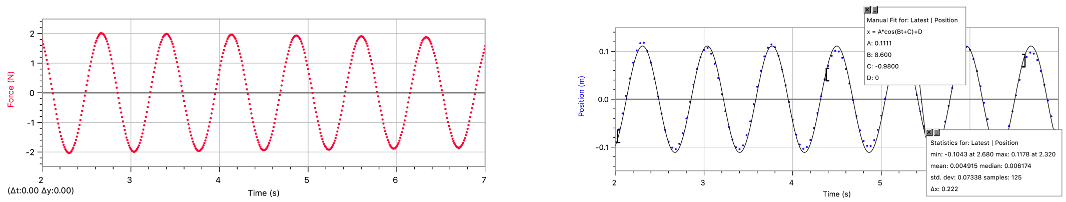


Figure 5: Time vs. Acceleration (fit line: $y = 7.301 \sin(8.6x + 2.19)$)

5.3 Net Force on the Oscillating Mass

5.3.1 Force and Displacement vs. Time



(a) Force vs. Time (b) Displacement vs. Time

Figure 6: Force and Displacement vs. Time

The wave of *Force vs. Time* is completely out of phase with *Displacement vs. Time*.

5.3.2 Force vs. Displacement

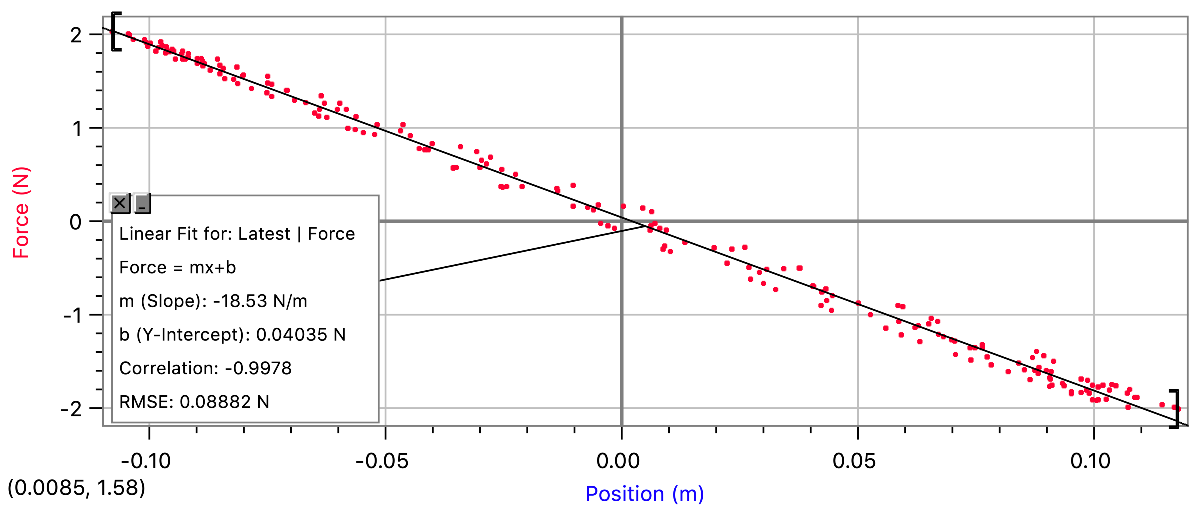


Figure 7: Force vs. Displacement

The meaning of this graph shows that as the spring being stretched towards negative displacement, the force that the spring goes against it increases. The slope is the spring constant of the spring, given that $\mathrm{N\,m^{-1}}$ is exactly the unit of spring constant.

5.3.3 Force vs. Acceleration

This graph shows the relationship between force and acceleration are linear. Also, it shows the increase of deacceleration causes the increase of force exerting on the negative axis, in our case means the spring stores energy at that moment.

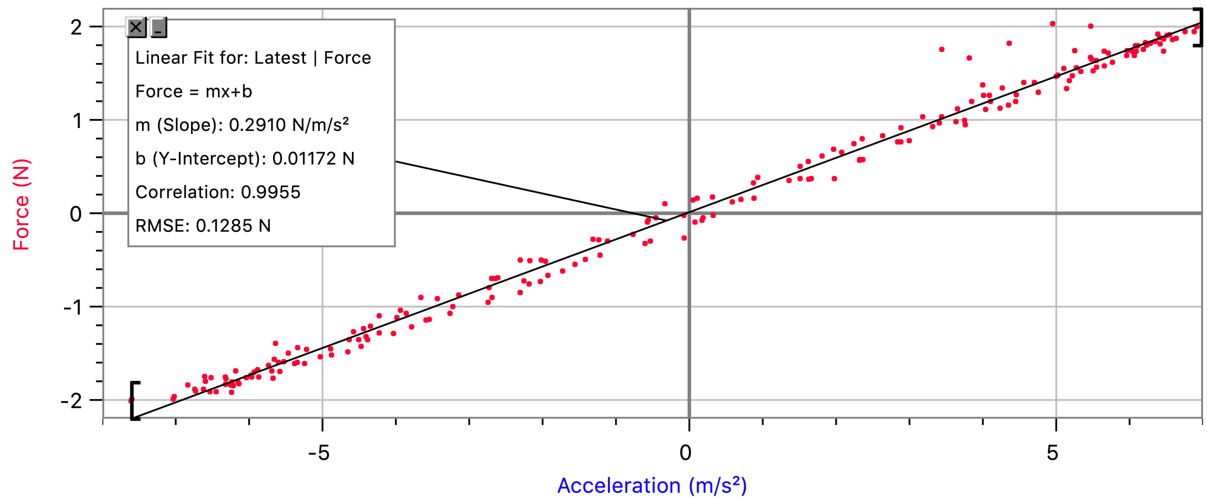


Figure 8: Force vs. Acceleration

The effective oscillating mass in our lab is 250g. The reason it is not the actual suspended mass value is because that the spring excerts force when the mass 'bounces'.

6 Conclusion

In this lab we discovers the true definition of Simple Harmonic Motion and we dive deep into it by manually fitting the wave function, as well as analyzing data using different dimentions. We discovered that there's linear relationship between *Force vs. Displacement* and *Force vs. Acceleration*. Such discover is valuable and essential for us to find that the slope of *Force vs. Displacement* graph is actually the spring constant.